Rank Minimization-Based Fast Image Completion

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Abstract

We propose an image completion algorithm using a rank minimization framework. Based on the low-rank property of an image, we formulate image completion as a low-rank matrix completion problem. Then, we solve the optimization problem efficiently using the augmented Lagrange multiplier (ALM) method. Experiments show that the proposed algorithm provides comparable or even higher image qualities than state-of-the-art approaches, while demanding significantly lower computational resources.

Keywords: Image inpainting, image completion, rank minimization, truncated nuclear norm minimization.

1. Introduction

Image completion is an image editing operation that replaces or fills regions in images with plausibly synthesized content. Image completion, which is also known as image inpainting, is used in a wide range of practical image processing applications, including the removal of objects in photographs and the recovery of partially damaged regions due to transmission errors in the encoded bitstream. In addition, it is an essential step in many graphics algorithms, *e.g.*, for reshuffling image contents or generating a clean background plate. Thus, It is important to develop efficient image completion algorithms, and considerable attempts have recently been made [1], [2].

Despite recent progress, providing high-quality images still remains an open challenge in image completion. This is because many algorithms require high-level understanding of the scene, which is difficult to detect automatically, as well as low-level cues via a local search. These algorithms may provide poor performance if the input image has less redundant information, *e.g.*, the scene is not front-parallel. Moreover, due to the non-convexity of the cost, those algorithms may converge to the local minima.

In this work, we propose a fast image completion algorithm using a rank minimization framework. Specifically, assuming the low-rank property of an input image, we formulate image completion as a rank minimization problem, and then solve it efficiently using the augmented Lagrange multiplier (ALM) method. Experimental results show that the proposed algorithm provides comparable or even higher image qualities than state-of-the-art approaches, while providing a substantial improvement in speed.

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2. Proposed Algorithm

We first assume that the images are regarded as approximately low-rank matrices, *i.e.*, the rank of the latent image $\mathbf{X}$ is low. We also assume that only a limited number of observations can be made in the input image $\mathbf{D}$ due to transmission errors in the encoded bitstream or objects in the input images. Thus, image completion can be formulated as the following rank minimization problem

$$\begin{aligned} &\underset{\mathbf{X}}{\text{minimize}} \quad \text{rank}(\mathbf{X}) \\ &\text{subject to} \quad \mathcal{P}_\Omega(\mathbf{X}) = \mathcal{P}_\Omega(\mathbf{D}) \end{aligned} \tag{1}$$

where $\mathbf{X} \in \mathbb{R}^{m \times n}$, and $\mathcal{P}_\Omega$ denotes a sampling operator in the observed region Ω [3], [4].

Note that it is intractable in practice to solve the optimization problem in (1) directly. Therefore, an approximate method via convex relaxation has been used in previous approaches [3], [4]. In this work, we employ the truncated nuclear norm to approximate the rank function in (1), because it is known to be a better approximation to the matrix rank [5]. Then, given the target rank r, the optimization problem in (1) can be rewritten as

$$\begin{aligned} &\underset{\mathbf{X}}{\text{minimize}} \quad \|\mathbf{X}\|_r \\ &\text{subject to} \quad \mathcal{P}_\Omega(\mathbf{X}) = \mathcal{P}_\Omega(\mathbf{D}) \end{aligned} \tag{2}$$

where $\|\mathbf{X}\|_r = \sum_{k=r+1}^{\min(m,n)} \sigma_k(\mathbf{X})$, and $\sigma_k(\mathbf{X})$ denotes the kth largest singular value of $\mathbf{X}$.

We solve the optimization problem in (2) by employing the ALM-based method [6]. To this end, by introducing the slack variable, we reformulate (2) as

$$\begin{aligned} &\underset{\mathbf{X},\mathbf{S}}{\text{minimize}} \quad \|\mathbf{X}\|_r \\ &\text{subject to} \quad \mathbf{X} + \mathbf{S} = \mathcal{P}_\Omega(\mathbf{D}) \\ &\qquad\qquad\quad \|\mathcal{P}_\Omega(\mathbf{D})\|_F \le \delta \end{aligned} \tag{3}$$

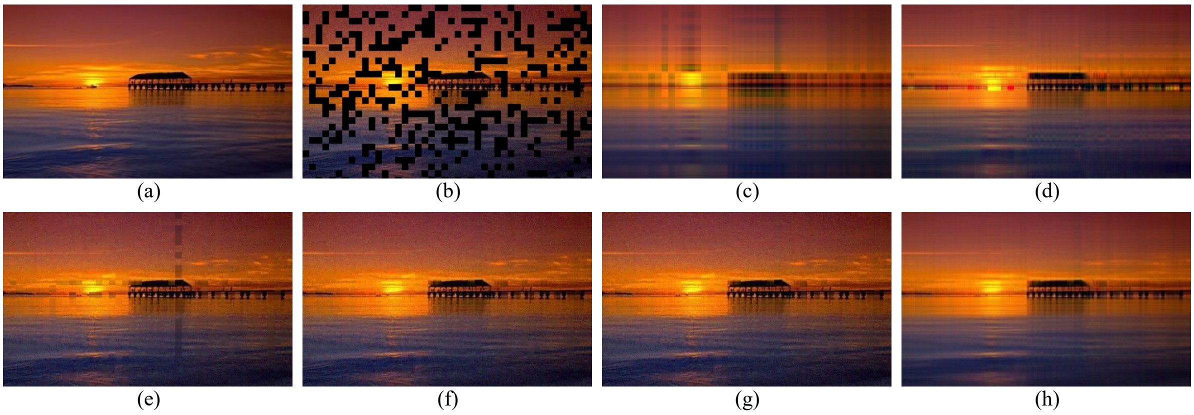

Fig. 1. Comparison of recovered images with missing blocks of a natural image using different methods when 70% of pixels are observed. (a) Original image [5] and (b) image with mask. Recovered images by (c) OptSpace [7] (25.23 dB), (d) LMaFit [8] (27.39 dB), (e) IALM-MC [9] (27.49 dB), (f) TNNR-ADMM [5] (27.81 dB), (g) IRNN-TNN [10] (27.85 dB), and (h) the proposed algorithm (30.78 dB).

where $\mathbf{S}$ is a matrix of slack variables, and $\|\mathbf{A}\|_F = \left(\sum_{ij}|A_{ij}|^2\right)^{1/2}$ denotes the Frobenius norm of a matrix. Also, δ is the noise level in the input image. Then, the augmented form of the optimization problem in (2) can be efficiently solved by the ALM method [6].

3. Experimental Results

We evaluate the performance of the proposed rank minimization-based image completion algorithm with those of state-of-the-art algorithms, *i.e.*, OptSpace [7], LMaFit [8], IALM-MC [9], TNNR-ADMM [5], and IRNN-TNN [10]. It is observed in [11] that, for a white noise with standard deviation σ, δ in (3) satisfies $\delta \leq \left(|\Omega| + \sqrt{8|\Omega|}\right)\sigma^2$, where $|\Omega|$ denotes the number of observations. Thus, we set $\delta = \left(|\Omega| + \sqrt{8|\Omega|}\right)^{1/2}\sigma$.

Note that missing information is spatially correlated, rather than randomly distributed in many real-world applications, *e.g.*, block missing in a compressed image. To simulate such a scenario faithfully, we evaluate the reconstruction performance for missing blocks. In this test, we set the block size as 8×8 to simulate the case of block missing in a compressed image using JPEG.

Fig. 1 compares reconstructed results on the test image when 70% of pixels are observed. We see that the proposed algorithm provides the best reconstruction performance in terms of both objective and subjective qualities. Specifically, on the test image in Fig. 1(a) with missing blocks in Fig. 1(b), the proposed algorithm provides 5.55, 3.39, 3.29, 2.97, and 2.93 dB higher PSNR values than OptSpace, LMaFit, IALM-MC, TNNR-ADMM, and IRNN-TNN, respectively. The actual execution times of OptSpace, LMaFit, IALM-MC, TNNR-ADMM, IRNN-TNN, and the proposed algorithm to obtain the results in Fig. 1 are 2.91, 0.303, 0.806, 55.0, 23.3, and 1.46 seconds, respectively. These results indicate that the proposed algorithm can achieve significant speedup over OptSpace, TNNR-ADMM, and IRNN-TNN and is comparable to IALM-MC, while providing substantially higher image qualities than all the conventional algorithms.

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